

**INTERNATIONAL CONFERENCE ON APPLIED MATHEMATICS  
AND NUMERICAL METHODS**

**Third Edition**

**Craiova, October 29-31, 2020**

**Sponsors:**



**MINISTERUL EDUCAȚIEI NAȚIONALE**



**UNIVERSITY  
OF  
CRAIOVA**

**Craiova, October 29-31, 2020**

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## Welcome to the third ICAMNM 2020

This year was one of multiple challenges for all of us, having to deal with the effects of the widespread of COVID - 19. Being devoted to organizing the third edition of our conference in the best manner possible, we took into account the safety of our participants and we decided to hold this event online. So we are very happy to welcome all the participants and invited guests to this online edition of ICAMNM and we would like to express our gratitude for their scientific contributions and for accommodating to the situation.

As always, the purpose of the conference is to bring together specialists in Mathematics and its applications, to present the latest scientific results, to exchange ideas and to identify possible future cooperation.

The program of the conference consists of invited talks given in plenary sessions, and presentations scheduled in the following sections of the conference:

- Applied Mathematics, Numerical Methods and Geometry;
- Differential Equations, Dynamical Systems, and Their Applications.

This book contains the abstracts of the presenters' contributions.

Special thanks to the members of the Scientific Committee who agreed to endorse all papers that will be presented.

Acknowledgements are also addressed to the Board of the University of Craiova, for their involvement in the organization of the conference, as well as to the generous financial support from our sponsor, the **Romanian Ministry of National Education**.

We wish to all the participants a successful joint work !

The Organizing Committee



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## Programme of Conference

### Thursday, October 29

**9:10-9:15** Opening Ceremony

**9:15-10:45** Plenary Lectures

9:15-9:45 **RĂDULESCU Vicențiu**, *Double phase problems with variable exponents and mixed regime*

9:45-10:15 **SOFONEA Mircea**, *Tykhonov triples, well-posedness results and applications*

10:15-10:45 **MOROȘANU Gheorghe**, *Two-parameter second-order differential equations in Hilbert spaces*

**10:45-11:00** Break

**11:00-12:30** Plenary Lectures

11:00-11:30 **MICU Sorin**, *Controllability properties of the wave equation with a second order memory term*

11:30-12:00 **MARIN Marin**, *The mixed problem in strain gradient thermoelasticity approached with the Lagrange identity*

12:00-12:30 **RIBEIRO Ana Margarida**, *Gradient flow formulations of discrete and continuous evolutionary models*

**12:30-14:00** Break

**14:00-16:40** Lectures on Sections

**16:40-17:00** Break

**17:00-20:00** Lectures on Sections

**Section 1. Applied Mathematics, Numerical Methods, and Geometry**

14:00-14:20 **CAN Engin**, *An investigation of three mathematical models on the Covid-19 pandemic: The case of Turkey*

14:20-14:40 **CONSTANTINESCU Dana**, *A general epidemiological model including the effect of social interaction*

14:40-15:00 **FLOREA Aurelia**, *A Mathematical Model of Infectious Disease Transmission*

15:00-15:20 **PAȘOL Vicențiu**, *Homomorphic encryption: theoretical and practical aspects*

15:20-15:40 **CHIRILĂ Adina**, *Spatial Behaviour of Thermoelasticity with Microtemperatures and Microconcentrations*

15:40-16:00 **DĂNEȚ Cristian**, *The study of a fourth order differential equation: existence, uniqueness and a dynamical system approach.*

16:00-16:20 **GRECU Luminița**, *Radial basis functions for solving the singular boundary integral equation of the compressible fluid flow around obstacles*

16:20-16:40 **ILIE Mihaela**, *Truthful Routing in Ad Hoc Mobile Wireless Networks Through Algorithmic Mechanism Design*

**16:40-17:00 Break**

17:00-17:20 **LAZUREANU Cristian**, *Stabilization of the  $-T$  system by an integrable deformation*

17:20-17:40 **MUNTEANU Florian**, *A Study of a Three-Dimensional Competitive Lotka–Volterra System*

17:40-18:00 **PAȘA Tatiana**, *Genetic algorithm for solving transportation problems on networks with one source and multiple sinks*

18:00-18:20 **BUNOIU Renata**, *Localization and multiplicity in the homogenization of nonlinear problems*

18:20-18:40 **PETRE Marin**, *Mathematical Modelling of the quenching process of 6061 aluminium alloy plates*

18:40-19:00 **ROCȘOREANU Carmen**, *Control of an economic dynamical system*

19:00-19:20 **POPESCU George**, *Hardy Type Inequalities, Rhyal Operators, Subsequences of Natural Numbers*

19:20-19:40 **ȘTERBEȚI Cătălin**, *On a Model for Populations with Age Structure*

19:40-20:00 **YERTAYEVA Sezim**, *Application of simplex method to management decision-making*



**Section 2. Differential Equations, Dynamical Systems, and Their Applications**

14:00-14:20 **CAZACU Cristian**, *Optimal Hardy-Rellich type inequalities*

14:20-14:40 **ZHANG Youpei**, *Kirchhoff problems with lack of compactness*

14:40-15:00 **BOUREANU Maria Magdalena**, *Weak solutions for general elliptic systems*

15:00-15:20 **ENACHE Cristian**, *United Arab Emirates A monotonicity property of the  $p$ -torsional rigidity*

15:20-15:40 **STANCU-DUMITRU Denisa**, *On a family of torsional creep problems in Finsler metrics*

15:40-16:00 **MIHĂILESCU Mihai**, *The Spectrum of the Relativistic Mean Curvature Operator*

16:00-16:20 **COSTEA Nicușor**, *Existence results for mixed hemivariational-like inequalities*

16:20-16:40 **AVCI Mustafa**, *The Regularization Method for Multivalued Elliptic PDEs with Variable Exponent*

**16:40-17:00 Break**

17:00-17:20 **GRECU Andrei**, *Fractional eigenvalue problems on  $R^N$*

17:20-17:40 **BABALIC Nicoleta-Corina**, *Soliton solutions for complex semidiscrete mKdV equation*

17:40-18:00 **IGNAT Liviu**, *Asymptotic behavior of solutions for local and nonlocal diffusion on metric graphs*

18:00-18:20 **TEMEREANCĂ Laurențiu Emanuel**, *Controllability properties of the heat equation with a memory term*

18:20-18:40 **GUȚU Valeriu**, *An algorithm for constructing Pythagoras Trees and other fractals (joint work with V. Glavan)*

18:40-19:00 **DIAMANDESCU Aurel**, *On the  $\Psi$ -asymptotic equivalence of the  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations*

19:00-19:20 **DIAMANDESCU Aurel**, *On the  $\Psi$ -asymptotic relationships between  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations*

19:20-19:40 **BĂLĂ Dumitru**, *Dynamical systems. Applications in economy*

**Friday, October 30****9:00-11:30 Plenary Lectures**

9:00-9:30 **CIOBAN Mitrofan**, *About the structure of extensions of topological groups with special remainders*

9:30-10:00 **MOROȘANU Costică**, *Numerical approximation and simulations for a nonlocal and nonlinear anisotropic reaction-diffusion model of second-order, supplied with non-homogeneous Cauchy-Neumann boundary conditions in 2D case*

10:00-10:30 **ION Stelian**, *Mathematical modelling of the plants effects on surface runoff and subsurface flow*

10:30-11:00 **CANGUL Ismail Naci**, *New Applications of Graphs*

11:00-11:30 **RAMOS CALLE Higinio**, *The use of block methods for solving singular boundary value problems*

**11:30-11:50 Break**

**11:50-13:10 Lectures on Sections**

**13:10-14:30 Break**

**14:30-16:30 Lectures on Sections**

**16:50-17:00 Break**

**17:00-19:00 Plenary Lectures**

**Section 1. Applied Mathematics, Numerical Methods and Geometry**

11:50-12:10 **KUMARI Pinki**, *Painlevé integrability and multisoliton solutions of a generalized KdV system*

12:10-12:30 **MAURYA Ashutosh**, *Fish Harvesting strategies with allee effect*

12:30-12:50 **MONDAL Hiya**, *Study of dynamo effect in thermal convection in the presence of rotation*

12:50-13:10 **PRIYADARSHI Anupam**, *Bigger perturbations enhance higher trophic levels biomass, increase transfer efficiency and may sustain for bigger plankton biodiversity*

**13:10-14:30 Break**

14:30-14:50 **PAȚIUC Vladimir**, *On Approximation of Higher-Order Derivatives*

14:50-15:10 **WOLDAREGAY Mesfin Mekuria**, *Uniformly convergent numerical scheme for singularly perturbed parabolic delay differential equations*

15:10-15:30 **DIMITROV Julian**, *An Approach for Local Sensitivity Analysis on Differentiable Manifold*

15:30-15:50 **BOBOESCU Remus**, *Algebraic definition of quaternions and relations with vectors in 3D space*

15:50-16:10 **POPESCU Marcela**, *On quaternionic product on some sets of hyperbolas*

16:10-16:30 **POPESCU Paul**, *On some hamiltonian forms of lagrangian dynamics*

16:30-16:50 **ANTIĆ Miroslava**, *Three-dimensional CR submanifolds of the nearly Kähler sphere  $S^6(1)$  that admit foliation by  $S^2(1)$*

**16:50-17:00 Break**

**Section 2. Differential Equations, Dynamical Systems, and Their Applications**

11:50-12:10 **CÎRSTEA Florica**, *Anisotropic elliptic equations with gradient-dependent lower order terms and  $L^1$  data*

12:10-12:30 **FĂRCĂȘEANU Maria**, *Sharp existence and classification results for nonlinear elliptic equations in  $R^N \setminus \{0\}$  with Hardy potential*

12:30-12:50 **TURAB Ali**, *On the positive solutions of a fractional thermostat model*

12:50-13:10 **GAUTAM Avinita**, *Impact of External Applied Currents in BVP Model*

**13:10-14:30 Break**

14:30-14:50 **VLADIMIRESCU Cristian**, *Stability for systems of 1-D coupled nonlinear oscillators*

14:50-15:10 **RUSU Galina**, *Two parameter singular perturbation problems for abstract second order differential equations with lipschitzian nonlinearities*

15:10-15:30 **COZMA Dumitru**, *Darboux integrability of a cubic system with one invariant straight line and one invariant cubic*

15:30-15:50 **BUCUR Liliana Maria**, *The asymptotic behavior of a dynamical process coming from the development in continuous fractions*

15:50-16:10 **IONESCU Adela**, *On Controlling the chaotic motion of the Chua dynamical system*

16:10-16:30 **YILMAZ Abdulkadir**, *Haar Wavelet Collocation Method for Linear First Order Stiff Differential Equations*

16:30-16:50 **VLAD Șerban**, *Asynchronous Combinational Systems*

**16:50- 17:00 Break**

**17:00- 19:00 Plenary Lectures**

17:00-17:30 **BALAN Vladimir** , *Geometric aspects of certain second order differential systems in particle physics*

17:30-18:00 **VÉLEZ-SANTIAGO Alejandro** , *The generalized anisotropic Wentzell problem with non-standard growth conditions*

18:00-18:30 **TARFULEA Nicolae** , *Boundary Conditions for Constrained Hyperbolic Systems of Partial Differential Equations*

18:30-19:00 **LEFEBVRE Mario** , *Analytical solution to an LQG homing problem in two dimensions*

**19:00 -19:15 Closing Ceremony**

## Plenary Lectures

### Geometric aspects of certain second order differential systems in particle physics

Vladimir BALAN, Nina KRYLOVA and Viktor RED'KOV

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**Abstract.** In the present work we consider the problem of the spin 1 particle with anomalous magnetic moment in an external Coulomb field, in non-relativistic approximation. The structural stability of the system is studied, and a class of solutions for the inverse problem is provided.

### New Applications of Graphs

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#### Abstract.

Recently a new topological graph invariant named as omega invariant has been defined by Delen and Cangul. It can be calculated for a given graph, too. This new invariant, defined for a given degree sequence, is in relation with the Euler characteristic and is directly related to the cyclomatic number of a graph. A new realizability criteria can be obtained and also helps to find algebraic, graph theoretical, number theoretical, topological and combinatorial properties of the realizations of a given degree sequence such as numbers of components, cyclicity, connectivity, multiple edges, chords, loops, cycles, pendant and support vertices, etc. Several applications of this invariant have been found. Here, we shall give new applications of graphs to some graph indices, matching number, connectivity, etc.

**2020 Mathematics Subject Classification Numbers:** 05C10, 05C30

**Keywords:** degree sequence, omega invariant, realizability, connectedness

Let  $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \dots, \Delta^{(a_\Delta)}\}$  be a non-decreasing set of positive integers. If there exists at least one graph  $G$  having  $D$  as its degree sequence, then we say that  $D$  is realizable and every  $G$  is called a realization of  $D$ . If  $D$  is realizable, then it has more than one realizations except a few cases. No results is given on the number of realizations or the numbers of components, loops, multiple edges, chords, bridges, etc. together with the connectedness, cyclicity, etc. of these realizations.

The omega invariant of a degree sequence is recently defined in [1]. It is related only to the degree sequence and gives several combinatorial and topological information on properties of the realizations of a degree sequence:

**DEFINITION 0.1.** Let  $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \dots, \Delta^{(a_\Delta)}\}$  be a set which also is the degree sequence of a graph  $G$ . The  $\Omega(G)$  of the graph  $G$  is defined only in terms of the degree sequence as

$$\begin{aligned}\Omega(G) &= a_3 + 2a_4 + 3a_5 + \dots + (\Delta - 2)a_\Delta - a_1 \\ &= \sum_{i=1}^{\Delta} (i - 2)a_i.\end{aligned}$$

In [2], some extremal problems on the number of components and loops have been investigated. Maximum number of components and loops in any realization of a given degree sequence is formulized. In [3], the cyclicity property of graphs have been studied by means of omega invariant. Acyclic, unicyclic, bicyclic, tricyclic, etc. graphs have been characterized by means of omega value of degree sequence. In [6], omega invariant is used to find some extremal values of graph theoretical indices. In [4], omega invariant is utilized to get results on matching number. In [5], a very important problem for degree sequences and their realizations have been studied and completely solved by means of omega invariant. We can decide which degree sequences can have connected realizations and which cannot have.

There are many open problems related to the degree sequences. For the realizability, several tests have already been given, the most well-known one being Havel-Hakimi process. Here we introduce a new and very practical test for the realizability. The relation  $\Omega(G) = 2(m - n)$  was given in [1] and is very useful in calculating  $\Omega(G)$  of a graph  $G$ . For any graph  $G$ ,  $\Omega(G)$  is even. Therefore if  $\Omega(G)$  is odd, then  $D$  is not realizable.

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### About the structure of extensions of topological groups with special remainders

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**Abstract.** Compactification and extensions of spaces arise in a wide variety of situations in mathematics and its applications (see [3–5]). Every considered space is assumed to be Tychonoff. If  $X$  is a dense subspace of a space  $B$ , then the space  $B$  is called an extension of  $X$  and the subspace  $Y = B \setminus X$  is called a *remainder* of the space  $X$ . If  $G$  is a dense subgroup of a topological group  $B$ , then the subspace  $Y = B \setminus G$  is called a *group-remainder* of  $G$ , and  $B$  is said to be a *group-extension* of  $G$ . Various kinds of completeness and cover properties were mentioned in [1]. The following theorem was proved in [2].

**Theorem 1.** *Suppose that  $G$  is a topological group,  $\delta G$  is an extension of  $G$ ,  $X = \delta G \setminus G$ ,  $\Phi$  is a compact subset of countable character in  $\delta G$  and  $Y = \Phi \setminus G$ . Then:*

1. *Either  $G$  is a paracompact feathered space or  $Y$  is a pseudocompact subspace.*
2. *If  $Y$  is not a dense subset of  $\Phi$ , then  $G$  is a paracompact feathered space.*
3. *If  $X$  is a pc-respectful space and  $G \cap \Phi \neq \emptyset$ , then  $G$  is a paracompact feathered space.*
4. *If  $X$  is a  $\mu$ -complete space and  $G \cap \Phi \neq \emptyset$ , then  $G$  is a paracompact feathered space.*
5. *If  $X$  is a normal isocompact space and  $G \cap \Phi \neq \emptyset$ , then  $G$  is a paracompact feathered space.*

This theorem is applied in the study of distinct images of topological groups.

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**Mathematical modelling of the plants effects on surface runoff and subsurface flow****Stelian ION, Dorin MARINESCU, and Stefan Gicu CRUCEANU**

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**Abstract.** The water flow on soil or subsurface in the presence of plants is a very complex phenomenon that is influenced by many physical-chemical-biological factors. To build up a good mathematical model, from theoretical and practical points of view, one needs to identify the key factors that control the intensity of the water-soil-plant interactions. In this talk we examine some mathematical models that take into account the variation in soil porosity, the characteristics of the cover plants and variation in the soil surface gradient.

**Analytical solution to an LQG homing problem in two dimensions****Mario LEFEBVRE**

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**Abstract.** An analytical solution is found to the problem of maximizing the time spent in the first quadrant by the two-dimensional diffusion process  $(X(t), Y(t))$ , where  $Y(t)$  is a controlled Brownian motion and  $X(t)$  is its integral. Moreover, we force the process to exit the first quadrant through the y-axis. This type of problem is known as LQG homing and is very difficult to solve explicitly, especially in two or more dimensions. Here the partial differential equation satisfied by the value function is solved by making use of the method of separation of variables. The exact solution is expressed as an infinite sum of Airy functions.

**The mixed problem in strain gradient thermoelasticity approached with the Lagrange identity****Marin MARIN**

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**Abstract.** In our paper we approach the theory of strain gradient thermoelasticity. First, we define the mixed initial-boundary values problem in this context. Then, with the help of an identity of Lagrange's type, we prove some theorems regarding the uniqueness of the solution of this mixed problem and two theorems with regards to the continuous dependence of solutions on loads and on initial data. We prove these qualitative results without recourse to any boundedness assumptions on the coefficients or to any laws of conservation of energy. Also, we do not impose restrictions on thermoelastic coefficients regarding their positive definition.

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### Controllability properties of the wave equation with a second order memory term

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**Abstract.** We study the internal null-controllability of a wave equation with memory in the principal part, defined on the one-dimensional torus. We assume that the control is acting on an open subset which is moving with a constant velocity. The main result of the paper shows that the equation is null controllable in a sufficiently large time and for initial data belonging to suitable Sobolev spaces. Its proof follows from a careful analysis of the spectrum associated with our problem and from the application of the classical moment method.

This talk is based on a joint work with Umberto Biccari.

### Numerical approximation and simulations for a nonlocal and nonlinear anisotropic reaction-diffusion model of second-order, supplied with non-homogeneous Cauchy-Neumann boundary conditions in 2D case

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**Abstract.** The paper is concerned with a qualitative analysis for a nonlinear second-order parabolic problem, endowed with a nonlinearity of cubic type as well as non-homogeneous Cauchy-Neumann boundary conditions. Under certain assumptions on the input data:  $f(t, x)$ ,  $w(t, x)$  and  $v_0(x)$ , we prove the existence, a priori estimates, regularity and uniqueness of a solution in the class  $W_p^{1,2}(Q)$ , extending the results already proven by other authors and making so the present mathematical model more capable to describe the complexity of certain classes of physical phenomena (image processing, for instance). Using the finite-difference method (*second-order* in time), an explicit approximation scheme in 2D is constructed for the proposed second-order PDE model. Numerical simulations illustrate the effectiveness of the mathematical model (the number of iterations in numerical scheme takes low values) in image restoration and segmentation.

**Keywords.** nonlinear anisotropic diffusion; qualitative properties of solutions; Leray-Schauder principle; finite difference method; numerical approximation scheme; image restoration and segmentation.

**Subject classification.** 35Bxx, 35K55, 35K60, 35Qxx, 65Nxx, 68U10.

### Two-parameter second-order differential equations in Hilbert spaces

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**Abstract.** Consider in a real Hilbert space the boundary-value problem

$$-\varepsilon u'' + \mu u' + Au + Bu = f(t), \quad 0 < t < T, \quad u(0) = u_0, \quad u(T) = 0,$$

where  $T > 0$  is a given time instant,  $\varepsilon, \mu$  are positive parameters,  $A$  is a maximal monotone operator, and  $B$  is a Lipschitz operator. We investigate the behavior of the solutions to this problem in two cases:

- (i)  $\mu > 0$  fixed and  $\varepsilon \rightarrow 0$ ,
- (ii)  $\varepsilon > 0$  fixed and  $\mu \rightarrow 0$ .

Notice that if  $\mu = 1$  and  $\varepsilon$  is a positive small parameter, the above problem is a Lions-type regularization of the Cauchy problem

$$u' + Au + Bu = f(t), \quad 0 < t < T, \quad u(0) = u_0,$$

which was recently studied by L. Barbu and G. Moroşanu [Commun. Contemp. Math. 19 (2017)]. Our abstract results are illustrated with examples related to the heat equation and the telegraph differential system.

### The use of block methods for solving singular boundary value problems

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**Abstract.** Singular boundary value problems appear frequently on the modellization of many physical phenomena as in catalytic diffusion reactions, chemical kinetics, thermal-explosion theory, or electro hydrodynamics, among others. The singular Lane-Endem equation is a typical kind of equation that models some of those problems. Unfortunately, the exact solutions can be rarely obtained. In this situation the block methods have been used extensively for approximating different kind of differential problems.

### Double phase problems with variable exponents and mixed regime

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**Abstract.** We consider a class of nonlinear double phase problems driven by differential operators with variable exponents. A feature of this problem is that it can fulfill a mixed regime subcritical-critical-supercritical, which is possible due to the presence of variable potentials. The main results establish sufficient conditions for the existence of nontrivial solutions.

### Gradient flow formulations of discrete and continuous evolutionary models

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**Abstract.** Biological evolution is a complex phenomena comprising several mechanisms, as natural selection, mutation or genetic-drift, among others. Having these mechanisms in mind, several models in evolutionary dynamics were developed by means of discrete time Markov chains, continuous time stochastic processes, and systems of ordinary differential equations. In this talk, I'll introduce gradient flow formulations to some classes of these three types of models relating the three perspectives. This is a joint work with F. Chalub, L. Monsaingeon, and M. Souza.

### Tykhonov triples, Well-posedness and Convergence Results Mircea SOFONEA

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**Abstract.** We present a unified theory of convergence results in the study of various nonlinear problems. To this end we introduce a new mathematical object, the so-called Tykhonov triple  $\mathcal{T} = (I, \Omega, \mathcal{C})$ , constructed by using a set of parameters  $I$ , a multivalued function  $\Omega$  and a set of sequences  $\mathcal{C}$ . Given an abstract problem  $\mathcal{P}$  and a Tykhonov triple  $\mathcal{T}$ , we introduce the notion of well-posedness of  $\mathcal{P}$  with respect to  $\mathcal{T}$  and provide several preliminary results in the framework of metric spaces. Then we show how these abstract results can be used to obtain various convergence results in the study of three relevant problems in reflexive Banach spaces: a nonlinear elliptic equation, a history-dependent equation, and a minimization problem. The results concern the continuous dependence of the solution with respect to the data and/or parameters as well as the convergence of the discrete solution to the solution of the continuous problem. Moreover, for each problem we provide an example arising from Solid or Contact Mechanics and give the mechanical interpretation of the corresponding convergence results.

### Boundary Conditions for Constrained Hyperbolic Systems of Partial Differential Equations Nicolae TARFULEA

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**Abstract.** Important mathematical models in science and technology are based on first order symmetric hyperbolic systems of differential equations whose solutions must satisfy certain constraints. When the models are restricted to bounded domains, the problem of well-posed, constraint-preserving boundary conditions arises naturally. However, for numerical solutions finding such boundary conditions may represent just a step in the right direction. Including the constraints as dynamical variables of a larger, unconstrained system associated to the original one could provide better numerical results, as the constraints are kept under control during evolution. One of the main goals of the talk is to present this idea in the case of constrained first order symmetric hyperbolic systems of differential equations subject to maximal nonnegative boundary conditions. As an example of application, a vector-valued wave equation with the constraint that the solution be divergence free is considered. Interestingly enough, on a smooth bounded domain, the set of constraint-preserving boundary conditions for this model problem involves the geometry of the boundary (i.e., the mean curvature). If the domain is polyhedral, then this set of boundary conditions belongs to the class of maximal nonnegative boundary conditions and the general theoretical result applies.

### The generalized anisotropic Wentzell problem with nonstandard growth conditions Alejandro VÉLEZ-SANTIAGO

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**Abstract.** We investigate the solvability and global regularity of a class of quasi-linear elliptic equations involving the anisotropic  $p(x)$ -Laplace operator with nonhomogeneous anisotropic Wentzell boundary conditions involving the anisotropic  $q(x)$ -Laplace-Beltrami operator. We first show existence and uniqueness of weak solutions for the elliptic problem, and moreover, we prove that such solutions are globally bounded. Key a priori estimates for the difference of weak solutions are provided, as well as Maximum principles and inverse positivity results. At the end, we establish a sort of "nonlinear anisotropic Fredholm alternative" for the corresponding anisotropic Wentzell problem of nonstandard structure.

## Applied Mathematics, Numerical Methods, and Geometry

### Three-dimensional CR submanifolds of the nearly Kähler sphere $S^6(1)$ that admit foliation by $S^2(1)$ Miroslava ANTIĆ

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**Abstract.** An almost complex manifold  $(\widetilde{M}, J)$  is nearly Kähler if its almost complex structure  $J$  satisfies  $(\widetilde{\nabla}_X J)X = 0$ , for any tangent section  $X$ . It is known that any nearly Kähler manifold is locally a product of three types of manifolds, one of them being six-dimensional nearly Kähler manifolds, and also that there are only four six-dimensional, homogeneous, nearly Kähler manifolds, sphere  $S^6(1)$  being one of them.

A submanifold  $M$  of an almost Hermitian manifold  $(\widetilde{M}, J)$  is a CR submanifold if admits a  $C^\infty$ -differentiable almost complex distribution  $D$  ( $JD \subset D$ ), such that its orthogonal complement  $D^\perp \subset TM$  is totally real, i.e.  $JD^\perp \subset TM^\perp$  and they represent the most natural generalization of the notions of almost complex and totally real submanifolds. Here, we are interested in three-dimensional CR submanifolds of the nearly Kähler, six-dimensional sphere  $S^6(1)$ . One of the first known families of the three dimensional minimal CR submanifolds in  $S^6(1)$  was introduced in [2] and [1]. We recall that a submanifold  $M$  of a Riemannian manifold  $(\widetilde{M}, g)$  is said to be ruled, if it admits a foliation with leaves that are totally geodesically immersed into  $\widetilde{M}$ . We investigate three dimensional CR submanifold of  $S^6(1)$  ruled by  $S^2(1)$  and give their explicit classification. In particular, we show that the examples given in [1] are of this type.

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### Dynamic systems. Applications in economy

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**Abstract.** In this paper, starting from recent data provided by the National Institute of Statistics, we analyze the tourism activity in Mehedinți County.

We apply the regression method and analyze some models. We also compare the economic results with those of previous years.

Also, in this paper we study the stability of dynamic systems with applications in economics. The stability study is done using the Leapunov function method.

The originality of the paper consists in the way we choose the mathematical model in case of regression and in the way we choose the Leapunov function in case of dynamic systems in which we analyze stability.

### Algebraic definition of quaternions and relations with vectors in 3D space that admit foliation by $S^2(1)$

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**Abstract.** For quaternions the scalar forms of imaginary parts  $(i, j, k)$  are associated with the unit vectors of the coordinate axes in the right three-dimensional Cartesian system  $(\vec{i}, \vec{j}, \vec{k})$ . The cross product of unit vectors of coordinate axes in three-dimensional space reproduces the multiplication table of imaginary quaternions without the main diagonal where the null vector appears. The null vector has a variable direction in the plane perpendicular

to unit vector whose square it represents. It is obvious that cross product of a vector with itself is not a correctly defined notion. In plan, as in space, we have the idea of an absolute Cartesian coordinate system. Towards this, the direction of a line is expressed by the slope. Thus the direction is expressed by a (real) number, being expressed relative to the absolute coordinate system. On any line in the plane with slope there is a perpendicular it has a slope . There is a relationship between the slope of the given line and the slope of the perpendicular line: The cross product between a vector and itself is not correctly defined, it requires two vectors. However, it could be done considering the equivalent directions of the same vector. The main relationships that define the characterization of vectors by quaternions are analyzed. The association between a vector and its direction is analyzed.

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**Localization and multiplicity in the homogenization of nonlinear problems****Renata BUNOIU**

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**Abstract.**

We propose a method for the localization of solutions for a class of nonlinear problems arising in the homogenization theory. The method combines concepts and results from the linear theory of PDEs, linear periodic homogenization theory, and nonlinear functional analysis. Particularly, we use the Moser-Harnack inequality, arguments of fixed point theory and Ekeland's variational principle. A significant gain in the homogenization theory of nonlinear problems is that our method makes possible the emergence of finitely or infinitely many solutions. Our study is motivated by real-world applications in physics, engineering and biology.

These results are based on the following joint work with Radu PRECUP, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj, Romania:

R. Bunoiu, R. Precup, Localization and multiplicity in the homogenization of nonlinear problems, *Adv. Non-linear Anal.* 9, (2020), 292–304.

**An investigation of three mathematical models on the Covid-19 pandemic: The case of Turkey**  
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**Abstract.** Mathematical modeling plays a major role in assessing, controlling, and forecasting potential outbreaks. This article consists of implementing mathematical forecasting models by Liang [14], Ma [16] and the curve fitting method with the least squares as a standard approach in regression analysis [20] and comparing their results from the Covid-19 outbreak data between March 10, 2020, and May 02, 2020, in Turkey, which the Turkish Ministry of Health announced. For this purpose, we presented an overview of three mathematical models and finally, demonstrated their applications using the Ministry of Health of Turkey's publicly reported data [19].

**MSC 2010:** 65C20, 65D10, 62J05**Keywords:** Mathematical model, Covid-19, coronavirus, Liang's model, Ma's model, curve fitting, the least squares method**References:** [14] Liang, K., Mathematical model of infection kinetics and its analysis for COVID-19, SARS and MERS. *Infection, Genetics and Evolution.* 82: (2020).

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**Spatial Behaviour of Thermoelasticity with Microtemperatures and Microconcentrations****Adina CHIRILĂ and Marin MARIN**

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**Abstract.** We consider a thermoelastic material with microtemperatures and microconcentrations, as in [1],[2],[4]. The mathematical model is represented by a system of partial differential equations with the coupling of the displacement, temperature, chemical potential, microconcentrations and microtemperatures fields. The processes of heat and mass diffusion play an important role in many engineering applications, such as satellite problems, manufacturing of integrated circuits or oil extractions [1].

We study the spatial behaviour in a prismatic cylinder occupied by an anisotropic and inhomogeneous material, following the approach from [3]. We impose final prescribed data that are proportional, but not identical, to their initial values. Moreover, we have zero body forces and zero lateral boundary conditions. The spatial behaviour is analysed in terms of some cross-sectional integrals of the solution that depend on the axial variable.

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### On quaternionic product on some sets of hyperbolas

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**Abstract.** The aim of this paper is to introduce some products, induced by the quaternionic product, on the set of equilateral hyperbolas. The study is a mix of elements from Euclidean and projective geometry. Some properties of these products are highlighted and are connected with some special numbers as roots or powers of the unit. Then we extend these products in a natural manner to oriented equilateral hyperbolas and to pairs of equilateral hyperbolas, using the algebra of octonions. Further, using an inversion, we extend these products to Bernoulli lemniscates and to Bernoulli q-lemniscates. Finally, using some isomorphisms, we extend these products to conics, and using an inversion, we extend these products to other curves. In order to highlight the importance of these studied products, the paper ends with some applications.

MSC: 51N20, 51N14, 11R52, 11R06.

Keywords: equilateral hyperbola; quaternion; product; projective geometry; octonion.

### A general epidemiological model including the effect of social interaction

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**Abstract.** A general model for the spread of contagious diseases is proposed. It takes into account susceptible, isolated, exposed, asymptomatic and symptomatic infected, hospitalized, deceased and recovered persons. We incorporate the social interaction in the model by considering the group of completely isolated people and the lockdown effect. Some other aspects, as the psychological reaction of the population when the number of infected people increases, are incorporated in a nonlinear incidence rate. The existence and stability of the equilibria is analysed and the reproduction number is computed. Various types of bifurcations are analysed and the results are interpreted from the epidemiological point of view. Numerical simulations complete the theoretical study.

## The study of a fourth order differential equation: existence, uniqueness and a dynamical system approach

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**Abstract.** This paper is concerned with the problem of existence and uniqueness of solutions for the semilinear fourth order differential equation  $u^{iv} - ku'' - a(x)u + c(x)f(u) = 0$ . We also give a dynamical system approach to the equation. We study the bifurcation of the system and show that the behaviour of the stationary points  $S(\alpha, 0, 0, 0)$  depend on the relation between the parameter  $k$  and  $a = f'(\alpha)$ .

**Keywords:** fourth order, dynamical system, bifurcation, Hopf bifurcation

**Mathematics Subject Classifications:** 35J35, 35J40, 49J27, 74K20, 37G10

## An Approach for Local Sensitivity Analysis on Differentiable Manifold

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**Abstract.** The models in applied sciences and engineering are characterized by significant errors of parameters. However, these models are presented by continuous dependences. A sensitivity analysis method is used to estimate the uncertainty in the numerical model corresponding to a continuous model. The uncertainty principle is a fundamental concept in the context of signal and image processing.

This paper discusses a method for valuation of the implicit functional dependencies - local functional dependences that presented by special case of differentiable manifolds with independent input variables and depended of them output variables. For optimal evaluation of the dependencies, we use relative distance in space of input parameters, calculations with semi-logarithmic derivatives ensuring accordance with the relative distance and we establish relevant properties. An application for evaluation of some technical tasks is described.

**M.S.C. 2010:** Primari 65C20, 26B12, 54E25; Secondary 49Q12, 68N30.

**Key words:** Numerical model, Relative distance, Hadamard product, Differentiable manifold, Condition for recoverability

## A Mathematical Model of Infectious Disease Transmission

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**Abstract.** The public health responses and questions create scenarios for every epidemic disease assumed that infection results in an immune response that protects individuals from future infections or illness for some amount of time. We know that the timescale of protection is a critical determinant of the future impact of the pathogen virus. The presence or absence of protective immunity due to infection or vaccination will affect future transmission and illness severity. We study a system of differential equations that models the population dynamics of an SIS vector transmitted disease with a pathogen virus, like SARS CoV-2 for example. Our model has been constructed to study the epidemiological trends of the disease.

**Keywords:** population dynamics, covid-19, dynamical systems, ODE.

**MSC 2010:** 37C75, 37N25, 34C23.



## Radial basis functions for solving the singular boundary integral equation of the compressible fluid flow around obstacles

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**Abstract.** In this paper we find a numerical solution for the Singular Boundary Integral Equation (SBIE), with sources distribution, of the compressible fluid flow around obstacles. The numerical solution is obtained by using linear boundary elements and radial basis functions for the unknown approximation and the Cauchy Principal Value for the treatment of singularities that appear. The proposed method is implemented into a computer code, made by using Mathcad programming language, and, for some particular cases, numerical solutions are found. An analytical checking of the computer code is also made, in order to validate the proposed approach.

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## Truthful Routing in Ad Hoc Mobile Wireless Networks Through Algorithmic Mechanism Design

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**Abstract.** Ad hoc networking links one or more devices together, wirelessly, for communication purposes. The linked devices can be mobile or not, which gives rise to mobile ad hoc networking(MANET).

The main challenges of MANETs are to avoid dropping packets by using reliable routes and, thus, increase throughput. We have found in the literature a secondary challenge in saving energy, and maximizing MANET life by load balancing routing instead of exhausting node energy one by one. In an ad hoc mobile network we can easily imagine a scenario where nodes wish to use the network but drop all packets that do not belong to themselves in order to conserve energy, computing resources etc. In the rest of this work we will label these nodes as *passive nodes*, and we will call the rest of the nodes *active nodes*.

In our literature review we found many works that set out to eliminate the problem of passive nodes and we cataloged them in three clusters:

- (1) *incentives for active nodes* : this approach has the potential of increasing throughput and decreasing the number of dropped packets. However, the technical solution for these incentives to be handed out and monitor node activity can be very costly in terms of communication and computation costs. This results in longer end-to-end times and incidentally very little throughput gain.
- (2) *actively isolate passive nodes* : this approach usually consist in some form of monitoring and detection of passive nodes which is generally communication intensive. Some approaches choose to drop any packets originated from confirmed passive nodes. However, since active nodes must spend energy to forward packets, there is high incentive to be as passive as possible without being detected.
- (3) *automatic active node detection* : this category uses bio-inspired algorithms that automatically *learn* to use predominantly active nodes. Basically these are more efficient than category (2) because they rely on returning packets and local computations instead of communications to monitor other nodes. However they do not punish passiveness. Coupled with more strain put on active nodes this results in an actual incentive to be passive. Another consequence of the stochastic nature of these protocols is the fact that unreliable routes might still be used from time to time resulting in more lost packets than a deterministic model.

We propose an algorithmic, mathematically defined, mechanism that will provide incentive for nodes to be active and truthfully collaborate while following their own selfish interests. Our mechanism is based on the well studied Vickrey Clarke Groves family of so-called *truthful* mechanisms that was first introduced in economy as a general auction mechanism. To apply this mechanism in networking, each node is expected to estimate a cost of relaying a message and it will always receive more than that in some form of currency. The chosen node to relay a message will always be the cheapest. The actual payment is calculated by estimating the cost to relay the message without the chosen node and this payment becomes higher. This approach is coupled with an underlying MANET routing algorithm to identify the cheapest paths. We will then analyze our formally and experimentally to compare with the existing state of the art to show similar performance for less computation and communication.

**Painlevé integrability and multisoliton solutions of a generalized KdV system****Pinki KUMARI, R.K. GUPTA, and Sachin KUMAR**

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**Abstract.** The integrability of a generalized KdV model, which has abundant physical applications in many fields, is investigated by employing Painlevé test. Eventually, we discover a new generalized P-type KdV model which is written as

$$\begin{aligned}u_t + au_{xxx} + buu_x + cvu_x &= 0, \\v_t + av_{xxx} + cvv_x + buv_x &= 0,\end{aligned}$$

in sense of WTC method. Subsequently, Hereman's simplified bilinear method is used to examine the integrability of the resulted model. As a result, multiple soliton solutions of newly discovered model are formally obtained.

**Keywords:** Generalized KdV system, Painlevé integrability, Multiple soliton, Hirota bilinear method

**Stabilization of the  $T$  system by an integrable deformation****Cristian LĂZUREANU and Cristiana CĂPLESCU**

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**Abstract.** Chaotic systems may have applications in various fields, such as secure communication, information processing, economics, biology. The chaotic behavior of such systems can be controlled using some control functions. A well-known chaotic system is the so-called  $T$  system [3].

Using the integrable deformation method [1], integrable deformations of the  $T$  system were obtained in [2]. As a consequence, in this paper we obtain control functions that stabilize the considered chaotic dynamics.

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**Fish Harvesting strategies with allee effect****Ashutosh MAURYA and Anupam PRIYADARSHI**

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**Abstract.** Fish harvesting is important for nutritious food availability in human diet. In this manuscript, a simple fish harvesting model is developed using logistic growth along with allee effect. We could done basic mathematical properties like equilibrium analysis and stability. For better understanding, we simulate the system with different constant harvesting rate and periodic harvesting rate. On comparing the result, we found periodic harvesting is economically beneficial than the constant harvesting in fish stock.

**Keywords:** Fish stock, Sine periodic harvesting, Economic beneficial

## Study of dynamo effect in thermal convection in the presence of rotation

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**Abstract.** Dynamo action is the process where the kinetic energy of the electrically conducting fluid in motion can generate magnetic field and current density in the absence of external magnetic field. To study dynamo effect, low-dimensional nonlinear model in Rayleigh-Bénard convection is constructed in the presence of rigid body rotation about vertical axis. In its geometry, a thin horizontal layer of viscous fluid is heated from below under an adverse temperature gradient. The critical values of reduced Rayleigh number ( $r$ ) at the dynamo onset vary significantly with thermal Prandtl number ( $Pr$ ), magnetic Prandtl number ( $Pm$ ) and Taylor number ( $Ta$ ). The different convective patterns of temperature field, magnetic field are explored near to dynamo onset. The model is also capable of capturing the disappearance and reappearance of the dynamo effect.

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## A Study of a Three-Dimensional Competitive Lotka–Volterra System

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**Abstract.** In this talk we will consider a community of three mutually competing species modeled by the Lotka–Volterra system:

$$\begin{cases} \dot{x}_i = x_i \left( b_i - \sum_{j=1}^3 a_{ij} x_j \right), & i = 1, 2, 3 \end{cases}$$

where  $x_i(t)$  is the population size of the  $i$ -th species at time  $t$ ,  $\dot{x}_i$  denote  $\frac{dx_i}{dt}$  and  $a_{ij}$ ,  $b_i$  are all strictly positive real numbers.

This system of ordinary differential equations has at most eight equilibrium points with positive components and represent a class of Kolmogorov systems. This kind of systems is widely used in the mathematical models for the dynamics of population, like predator-prey models or different models for spread of diseases.

A qualitative analysis of this Lotka–Volterra system based on dynamical systems theory will be performed, by studying the local behavior in equilibrium points and obtaining local dynamics properties.

**AMS Subject Classification (2010):** 37C25, 37D20, 37G35, 92D25.

**Key words:** Dynamical systems, Lotka–Volterra systems, stability.

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### Genetic algorithm for solving transportation problems on networks with one source and multiple sinks

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**Abstract.** In this paper we propose a genetic algorithm for solving the non-linear transportation problem on a network with multiple sinks and concave piecewise cost functions. We prove that the complexity of one iteration of the algorithm is  $O(n^2)$  and the algorithm converges to a local optimum solution. We show that the algorithm can be used to solve large-scale problems and present the implementation and several testing examples of the algorithm using Wolfram language.

### Homomorphic encryption: theoretical and practical aspects

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**Abstract.** In this talk we shall give a short overview of what is homomorphic encryption, its history and developments. We shall then present some of the practical applications that this field can achieve. We also present aspects of security analysis and some recent quantum attacks on generic homomorphic encryption schemes. If time permits, we shall present the architecture that shall be implemented in a contact tracing protocol that employs homomorphic encryption.

### On Approximation of Higher-Order Derivatives

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**Abstract.** We consider the problem of numerical differentiation frequently appearing in numerical analysis, i.e. the problem that describes the methods for approximating the derivatives of different orders of mathematical functions using known values of these functions. In the present study we generate a finite difference schemes that approximate the values of higher-order derivatives and then prove the correctness of the obtained formulas. The method of undefined coefficients has been used as a basis as well as the Taylor series expansion that provides a systematic way of deriving approximation to higher order derivatives of any order (provided that the function is smooth enough). It is assumed for simplicity that the grid is uniformly spaced.

### Mathematical Modelling of the quenching process of 6061 aluminium alloy plates

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**Abstract.** In recent decades, due to the increase in computing power, mathematical modelling has experienced a fulminant development in almost all areas. The aluminium industry is one of these areas. One of the main processes for improving the properties of certain aluminium alloys is the solution heat treatment and quenching process. The most common quenchant used for aluminium alloys is water. The main advantage of using a water quenchant is that water can provide the rapid quenching. By considering the temperature dependence of the thermo-physical

properties, the non-linear thermo-mechanical direct coupled analysis of the quenching process for a 6061 aluminium alloy plate was achieved. The structural stress due to solid thermal effects were studied by using ANSYS finite element software. The quenching rate, which determines the plate deformation after quenching, was estimated and validated on independent equipment for the research of aluminium alloy quenching process. The developed mathematical model serves as a tool by simulation of various scenarios that may occur in the industrial process.

**Key words:** ANSYS, quenching, 6061 aluminium alloy, quenching rate

**Bigger perturbations enhance higher trophic levels biomass, increase transfer efficiency and may sustain for bigger plankton biodiversity**

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**Abstract.** Recent observations have revealed that intermittent spatial distributions of phytoplankton are ubiquitously present at micro-scale ( $< 1 \text{ mm}$ ) and display consistently high degrees of  $mm$  scale patchiness from estuarine to open ocean waters. Plankton models are conventionally developed based on the mean-field approach in which impacts of second and higher central-moments approximations are ignored. The conventional modeling may be suitable for meso- or bigger scale ( $km$ ) but it is inappropriate for micro-scale ( $\leq 1 \text{ mm}$ ). A new modelling approach called *closure modelling* is developed to account spatial intermittency of phytoplankton using Reynold's decomposition and retaining higher central moments approximations in Taylor series expansion. The interactions of nutrient (N), phytoplankton (P), zooplankton (Z) and detritus (D) is modelled into NPZD model based on the closure modelling approach accounting both mean and fluctuating parts of these plankton variables. We investigate how perturbations affect transfer efficiency (a proxy for energy transfer within trophic levels) and biomasses of these planktonic variables. extensive numerical simulations are carried out to understand the qualitative behavior of plankton dynamics and effects of perturbations. The transfer efficiency increases with perturbations and hence zooplankton biomass enhanced with bigger perturbations. On comparison of conventional modelling approach (without perturbations) and closure modelling approach (in presence of perturbations), we observed that fluctuations or perturbations positively impact plankton dynamics as transfer efficiency increases and biomasses of higher trophic levels enhance. Based on our study, we hypothesize that "*higher spatial intermittency of phytoplankton distributions i.e. strong dominance of intermittent spatial distributions over spatial mean phytoplankton biomass enhance transfer efficiency, support higher trophic levels of plankton ecosystems and may sustain bigger biodiversity*".

**Keywords:** Perturbations, Transfer Efficiency, Closure Modelling approach, Central-moments approximations

**Hardy Type Inequalities, Rhaly Operators, Subsequences of Natural Numbers**

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**Abstract.** The starting point for Analysis is the sequence of natural numbers. We will show that two independently developed topics - the famous Hardy type inequalities and Terraced matrices or Rhaly operator - these two topics could be reduced somehow to the starting point - namely - subsequences of natural numbers. Properties of Rhaly operators as being bounded or compact, are reduced to behavior of subsequences of natural numbers, actually subseries of the simple harmonic series  $\sum \frac{1}{n}$ .

**Control of an economic dynamical system**

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**Abstract.** In this work a control variable was added to a well-known three-dimensional financial system, in order to obtain new types of stable behavior. The stability of equilibria is established. All the Hopf singularities were analyzed and the center manifolds were computed. We found stable limit cycles, saddle type limit cycles and isolated periodic solutions corresponding to a center equilibrium. A double-Hopf degenerated bifurcation is also emphasized.

### On a Model for Populations with Age Structure

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**Abstract.** In this paper we study a linear continuous model describing age structure into a dynamics of one sex population, related with the McKendrick model. In this model McKendrick assumes that the female population can be described by a function of two variables, age and time. Using the method of characteristics and Laplace transform, we find the function representing the number of births in unit time  $t$  and the total population size in some particular cases.

### Application of simplex method to management decision-making

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**Abstract.** Creating an effective production plan is a must for every company. However, there are many obstacles to take into consideration. First of all, it is important for the management to find the ways to maximize profits. This can only be done by taking into account all the constraints. If there's only one limiting factor and not so many differences in products, production plan can be easily conducted manually. Unfortunately, usually this is not the case as in practice every company has thousands of different stock keeping units, and several limiting factors tend to appear simultaneously. As a consequence, a decision-making process becomes way more complicated. In this case, application of linear programming technique would be the most suitable option. Linear programming is a mathematical method that is used to solve optimization problems. Simplex method is a part of linear programming and being largely applied in data sciences. However, this method can also be handy for management decision-making to maximize profits (and/or shorten costs). In this report we justify the advantage of using simplex method by providing a problem solving example.

**Key words:** management decision-making, limiting factors, production plan, simplex method

### Uniformly convergent numerical scheme for singularly perturbed parabolic delay differential equations

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**Abstract.** In this paper, exponentially fitted finite difference scheme is developed for solving singularly perturbed parabolic delay partial differential equations having small delay on the spatial variable. The term with the delay is approximated using Taylor series approximation. The resulting singularly perturbed parabolic partial differential equation is treated using implicit Euler method in the temporal discretization with exponentially fitted operator finite difference method in the spatial discretization. The parameter uniform convergence analysis has been carried out with the order of convergence one. Test examples and numerical results are considered to validate the theoretical analysis of the scheme.

**AMS 2010 Subject classification:** 65M06, 65M12, 65M15.

**Key words:** Delay differential equations, Exponentially fitted scheme, Uniform convergence.



# Differential Equations, Dynamical Systems, and Their Applications

## The Regularization Method for Multivalued Elliptic PDEs with Variable Exponent

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**Abstract.** In the present paper, we seek solutions to a multivalued nonlinear elliptic equation under homogeneous Dirichlet boundary condition settled in variable Lebesgue spaces. The main tools are the monotone operator theory [1], and the Regularization method [3].

We are concerned with the following multivalued nonlinear elliptic equation

$$-\nabla \cdot a(x, \nabla u) \ni \nabla \cdot g, \quad u|_{\partial\Omega} = 0, \quad u \in W_0^{1,p(x)}(\Omega), \quad (1)$$

where  $\Omega$  is a bounded domain in  $\mathbb{R}^N$  ( $N \geq 3$ ) with smooth boundary  $\partial\Omega$ ,  $a : \Omega \times \mathbb{R}^N \rightarrow \mathbb{R}^N$  is a multivalued function, and  $g \in L^{p'(x)}(\Omega)^N$ .

### References:

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## The soliton solutions for semidiscrete complex mKdV equation

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**Abstract.** The semidiscrete complex modified Korteweg de Vries equation (semidiscrete cmKdV), which is the second member of the semidiscrete nonlinear Schrodinger hierarchy (Ablowitz-Ladik hierarchy), is solved using the Hirota bilinear formalism. Starting from the semidiscrete form of cmKdV, proposed by Ablowitz and Ladik:

$$\frac{d}{dt}\phi_n = (1 + \alpha|\phi_n|^2)(\phi_{n+1} - \phi_{n-1}),$$

we construct the bilinear form:

$$\begin{aligned} D_t G \cdot F &= G_{n+1}F_{n-1} - G_{n-1}F_{n+1} \\ F^2 + \alpha|G|^2 &= F_{n+1}F_{n-1} \end{aligned}$$

and build the multi-soliton solutions. The complete integrability of semidiscrete cmKdV is proven and results are discussed and plotted.

**Acknowledgements:** This work was supported by the grant POCU380/6/13/123990, co-financed by the European Social Fund within the Sectorial Operational Program Human Capital 2014 – 2020.

## Weak solutions for general elliptic systems

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**Abstract.** We are concerned with the weak solvability of a class of anisotropic elliptic systems involving variable exponents and Leray-Lions type operators. To illustrate the generality of the main theorem presented in this talk, we give several examples of systems which represent particular cases of our initial system.

**Acknowledgements.** The author was partially supported by the GDRI EcoMath Project "Qualitative study of nonlinear PDE's" and by the Horizon2020-2017-RISE-777911 project.

**The asymptotic behavior of a dynamical process coming from the development in continuous fractions**  
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**Abstract.** The aim of this paper is the study of a dynamical process generated by a sequence of maps:

$$x_{n+1} = f_n(x_n)$$

Where  $f_n : (0, \infty) \rightarrow (0, \infty)$ ,  $f_n(x) = \frac{c_n}{1+x}$  for all  $n \in N$  and  $(c_n)_n$  is a sequence of positive numbers.

This process is generated similarly to continuous fractions development.

A continued fraction is an expression obtained through an iterative process of representing a number as the sum of its integer part and the inverse of another number, then writing this other number as the sum of its integer part and another inverse, and so on. In a finite continued fraction (or terminated continued fraction), the iteration is terminated after finitely many steps by using an integer in lieu of another continued fraction. In contrast, an infinite continued fraction is an infinite expression. In either case, all integers in the sequence, other than the first, must be positive. The integers are called the coefficients or terms of the continued fraction.

We will study the pre-equilibrium points for this process, the attraction basins and the stability.

**Optimal Hardy-Rellich type inequalities**

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**Abstract.**

In this talk we discuss both Hardy and Hardy-Rellich inequalities which establish useful properties for differential operators with singular potentials, for instance for non-symmetric expressions of the form  $-\Delta + \lambda \frac{x \cdot \nabla}{|x|^2} + \frac{\beta}{|x|^2}$ ,  $\lambda, \beta \in \mathbb{R}$ . In particular, we discuss some recent results by Gesztesy and Littlejohn (2018) and present some new extensions to weighted Hardy-Rellich type inequalities. We analyze the situations for both the radial operators  $\nabla_r, \Delta_r$  and the non-radial operators  $\nabla, \Delta$ , respectively. We focus on the best constants and the existence/nonexistence of minimizers in the energy space.

**Acknowledgements:** This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

**Anisotropic elliptic equations with gradient-dependent lower order terms and  $L^1$  data**

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**Abstract.** We prove the existence of a weak solution to the Dirichlet problem

$$\begin{cases} \mathcal{A}u - \mathfrak{B}u + \Phi(x, u, \nabla u) = f & \text{in } \Omega, \\ u \in W_0^{1, \vec{p}}(\Omega), \quad \Phi(x, u, \nabla u) \in L^1(\Omega), \end{cases}$$

where  $\Omega$  is a bounded open subset of  $\mathbb{R}^N$  with  $N \geq 2$ . Here  $\mathcal{A}$  is a divergence-form nonlinear anisotropic operator, the prototype of which is  $-\Delta_{\vec{p}} u = -\sum_{j=1}^N \partial_j (|\partial_j u|^{p_j-2} \partial_j u)$  with  $p_j > 1$  for all  $1 \leq j \leq N$  and  $\sum_{j=1}^N (1/p_j) > 1$ . We make suitable assumptions on the operator  $\mathfrak{B}$  so that  $\mathcal{A} - \mathfrak{B}$  is coercive from  $W_0^{1, \vec{p}}(\Omega)$  into  $W^{-1, \vec{p}'}(\Omega)$  and it maps bounded sets into bounded sets. We allow the presence of a gradient-dependent lower order term  $\Phi$ , that grows like  $\sum_{j=1}^N |\partial_j u|^{p_j}$ , and any function  $f \in L^1(\Omega)$ . This is joint work with Barbara Brandolini (University of Naples).

**Existence results for mixed hemivariational-like inequalities****Nicușor COSTEA**

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**Abstract.** In this talk we present some existence results for a nonstandard class of hemivariational-like inequalities involving set-valued maps and a (nonlinear) bifunction. The presence of the set-valued map ensures that not one, but two types of solution can be defined, while the presence of the bifunction does not allow the inequality to be written equivalently as an inclusion, making nonsmooth critical point theory unavailable. Using topological methods we are able to prove that our inequality possesses at least one solution (strong solution) provided the set-valued map is upper semicontinuous (lower semicontinuous, respectively). One of the main points of interest in our approach is that we are able to prove the existence of at least one solution even if the KKM technique fails, therefore some of our results are new even in the particular case when the problem reduces to a classical hemivariational, or variational inequality.

A nontrivial example for which our theoretical results are valid is also discussed. More precisely, we consider a differential inclusion involving the  $\Phi$ -Laplacian and mixed boundary conditions whose variational formulation leads to a hemivariational-like inequality.

**Acknowledgements:** This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

**Darboux integrability of a cubic system with one invariant straight line and one invariant cubic****Dumitru COZMA**

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**Abstract.** We consider the cubic system of differential equations

$$\dot{x} = y + p_2(x, y) + p_3(x, y), \quad \dot{y} = -x + q_2(x, y) + q_3(x, y), \quad (1)$$

where  $p_j(x, y), q_j(x, y) \in \mathbb{R}[\curvearrowright, \curvearrowleft]$  are homogeneous polynomials of degree  $j$ . The origin  $O(0, 0)$  is a singular point of a center or a focus type for (1). The problem of the center is still open for cubic systems.

The conditions under which the cubic system (1) is Darboux integrable, having first integrals or integrating factors of the form  $l_1^{\alpha_1} l_2^{\alpha_2} \Phi^{\alpha_3}$ ,  $\alpha_j \in \mathbb{C}$ , where  $l_1 = 0, l_2 = 0$  are invariant straight lines and  $\Phi = 0$  is an irreducible invariant conic (invariant cubic), were found in [1] ([2]).

The problem we consider in this talk is the following: find the cubic differential systems (1), which are Darboux integrable, having two algebraic solutions  $1 + a_1x + b_1y = 0, x^2 + y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 = 0$ , where  $(a_{30}, a_{21}, a_{12}, a_{03}) \neq 0, (a_1, b_1) \neq 0$  and  $a_{ij}, a_1, b_1 \in \mathbb{R}$ .

If the differential system (1) has a singular point of a center or a focus type and we are able to construct a Darboux first integral or a Darboux integrating factor with algebraic solutions, then the singular point is a center.

**References:**

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- [2]. Cozma D., Dascalescu A. Integrability conditions for a class of cubic differential systems with a bundle of two invariant straight lines and one invariant cubic. Bul. Acad. de Șt. a Republicii Moldova. Matematica. 2018, vol. 86, no.1, 120–138.

**On the  $\Psi$ -asymptotic equivalence of the  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations****Aurel DIAMANDESCU**

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**Abstract.** Using Schauder - Tychonoff fixed point theorem and the technique of Kronecker product of matrices, there are proved existence results for  $\Psi$ -asymptotic equivalence of the  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations.

**On the  $\Psi$ -asymptotic relationships between  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations****Aurel DIAMANDESCU**

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**Abstract.** Using Schauder - Tychonoff fixed point theorem and the technique of variation of constants formula combined with Kronecker product of matrices, there are proved existence results for  $\Psi$ -asymptotic equivalence of the  $\Psi$ -bounded solutions of two Lyapunov matrix differential equations.

**A monotonicity property of the  $p$ -torsional rigidity****Cristian ENACHE**Department of Mathematics and Statistics, American University of Sharjah,  
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**Abstract.** For a bounded domain  $\Omega \subset \mathbb{R}^N$ ,  $N \geq 2$  and a real number  $p > 1$ , we denote by  $u_p$  the  $p$ -torsion function on  $\Omega$ , that is the solution of the torsional creep problem  $\Delta_p u = -1$  in  $\Omega$ ,  $u = 0$  on  $\partial\Omega$ , where  $\Delta_p := \operatorname{div}(|\nabla u|^{p-2} \nabla u)$  is the  $p$ -Laplace operator. In this talk we are going to present some monotonicity properties for the  $p$ -torsional rigidity on  $\Omega$ , defined as  $T_p(\Omega) := \int_{\Omega} u_p dx$ . More precisely, we first show that there exists  $T \in (0, 1]$  such that for each open, bounded, convex domain  $\Omega \subset \mathbb{R}^N$ , with smooth boundary and  $\delta(\Omega) \leq T$ , where  $\delta(\Omega)$  represent the average integral on  $\Omega$  of the distance function to the boundary of  $\Omega$ , the function  $p \rightarrow T(p; \Omega) := |\Omega|^{p-1} T_p(\Omega)^{1-p}$  is increasing on  $(1, \infty)$ . Moreover, we also show that for any real number  $s > T$ , there exists an open, bounded, convex domain  $\Omega \subset \mathbb{R}^N$ , with smooth boundary and  $\delta(\Omega) = s$ , such that the function  $p \rightarrow T(p; \Omega) := |\Omega|^{p-1} T_p(\Omega)^{1-p}$  is not a monotone function of  $p \in (1, \infty)$ . Finally, we use this result to get a new variational characterization of  $T_p(\Omega)$  in the case when  $\delta(\Omega)$  is small enough.

**Keywords:** distance function to the boundary; torsional rigidity;  $p$ -Laplacian.**2010 Mathematics Subject Classification:** 35P30; 47J10; 49R05; 49J40; 58C40.**References**

- [1] C. Enache, M. Mihailescu, *A Monotonicity Property of the  $p$ -Torsional Rigidity*, submitted.

**Sharp existence and classification results for nonlinear elliptic equations in  $\mathbb{R}^N \setminus \{0\}$  with Hardy potential****Maria FĂRCĂȘEANU**

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**Abstract.** We reveal the structure and asymptotic behaviour near zero and infinity of all positive solutions for the nonlinear elliptic equation with Hardy potential  $(*) -\Delta u - \frac{\lambda}{|x|^2} u + |x|^\theta u^q = 0$  in  $\mathbb{R}^N \setminus \{0\}$  ( $N \geq 3$ ), where  $q > 1$ ,  $\theta \in \mathbb{R}$  and  $\lambda \in \mathbb{R}$  are arbitrary. We provide the sharp range of the parameters such that there exist positive solutions of  $(*)$  in  $\mathbb{R}^N \setminus \{0\}$ . We show that equation  $(*)$  has either a unique solution or infinitely many solutions or no positive solutions. This is joint work with Florica Cîrstea.

**Impact of External Applied Currents in BVP Model****Avinita GAUTAM and Anupam PRIYADARSHI**

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**Abstract.** Dynamics of neurons and its mathematical modeling are very interesting area of research. There are several researches on behavior of neurons, neurological disorders in order to understand the dynamics of neurons mathematically and computationally. In this manuscript, we have analyzed the impact of different externally applied currents for BVP model. Neurons are in resting-state when the system is stable. Hodgkin-Huxley model follows all-or-none law but the BVP model does not. In the BVP model, there is an intermediate phase where no spike forms. But sufficiently large input generates the spikes. When constant current has been applied then the system shows stable and periodic behavior for specific current values. If sinusoidal and wavelet form of external applied current is injected then we have observed that continuous firing. Numerical simulation has been performed to understand the bifurcation characteristics of the BVP model.

**Keywords:** Neurons, BVP Model, External Current, Periodic, Bifurcation

### Fractional eigenvalue problems on $\mathbb{R}^N$

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**Abstract.** Let  $N \geq 2$  be an integer. For each real number  $s \in (0, 1)$  we denote by  $(-\Delta)^s$  the corresponding fractional Laplace operator. First, we investigate the eigenvalue problem  $(-\Delta)^s u = \lambda V(x)u$  on  $\mathbb{R}^N$ , where  $V : \mathbb{R}^N \rightarrow \mathbb{R}$  is a given function. Under suitable conditions imposed on  $V$  we show the existence of an unbounded, increasing sequence of positive eigenvalues. Next, we perturb the above eigenvalue problem with a fractional  $(t, p)$ -Laplace operator, when  $t \in (0, 1)$  and  $p \in (1, \infty)$  are such that  $t < s$  and  $s - N/2 = t - N/p$ . We show that when the function  $V$  is nonnegative on  $\mathbb{R}^N$ , the set of eigenvalues of the perturbed eigenvalue problem is exactly the unbounded interval  $(\lambda_1, \infty)$ , where  $\lambda_1$  stands for the first eigenvalue of the initial eigenvalue problem.

**Acknowledgements:** This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

### An algorithm for constructing Pythagoras Trees and other fractals

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**Abstract.** Many fractals are associated with the attractors of hyperbolic Iterated Function Systems (IFSs). At the same time, a series of papers are devoted to the question: What kinds of compacta can (or can not) serve as attractors of hyperbolic IFSs?

M. Barnsley (1988) introduced the notion of Iterated Function System with condensation (IFSC). This idea is related to a finite collection of contractions, accompanied by a constant compact-valued mapping. It led to new fractals such as the well-known fractal The Pythagoras Tree.

In a paper of authors (2011) a question was formulated: Is it possible to reduce the construction of the attractor of a hyperbolic IFS with condensation to the construction of the attractor of an appropriate hyperbolic IFS in a general case, for example, in the case of the Pythagoras Tree?

Some answers to this question are already known. It is proved (V. Guțu, 2013, 2019) that any finite union of convex compacta in  $\mathbb{R}^n$  may be represented as the attractor of a hyperbolic IFS. Moreover, if such a union is added to a hyperbolic IFS as the condensation set, then the attractor of this IFS with condensation can be represented as the attractor of an appropriate hyperbolic IFS.

Based on these results we show how to construct the corresponding hyperbolic IFS for such an IFS with condensation in the plane.

We extend the Random Iteration Algorithm (M. Barnsley, 1988) to construct the attractor of an IFS with condensation, whose condensation set is a union of convex compacta in the plane, as the attractor of an appropriate hyperbolic IFS. This algorithm is illustrated on a hyperbolic IFS with condensation, whose attractor is the Pythagoras Tree. This fractal can be obtained using only five contractions.

Although there are many computer programmes for constructing fractals, this algorithm may be also of interest, due to a large class of IFSs with condensation, for which it can be applied. As a consequence, a large class of new fractals can be constructed using the computer.

This work was partially supported by the SCSTD ASM Project 15.817.02.26F.

**Asymptotic behavior of solutions for local and nonlocal diffusion on metric graphs**

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**Abstract.** In this talk we present some recent result about the long time behavior of the solutions for some diffusion processes on a metric graph. We prove that when time is large the solution behaves more and more like a gaussian profile on the infinite edges. This is a joint work with Julio D. Rossi (Buenos Aires) and Angel San Antolin (Alicante).

**On Controlling the chaotic motion of Chua dynamical system**

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**Abstract.** The chaotic behavior is observed in practical applications of many fields, from engineering to biology and economics. The problems and methods of controlling the chaos are subject of intensive interest, because of their practical importance. The present paper approaches the control of chaotic behavior for a well-known dynamical system, the Chua system. Using a simple linear controller, the system is driven to a stable state using the Lyapunov function method.

**AMS Subject Classification (2010):** 93C40, 93C15, 93D15, 93B18

**Key words:** adaptive control; stabilization by feedback; Chua system; linearization.

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- [3] L. O. Chua, *Nonlinear Circuits*, IEEE Transactions on Circuits and Systems, vol 31, no 1, 1984, pp 69-87, Centennial issue.

**The Spectrum of the Relativistic Mean Curvature Operator**

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**Abstract.** We show that the spectrum of the relativistic mean curvature operator on a bounded domain  $\Omega \subset \mathbb{R}^N$  ( $N \geq 1$ ) having smooth boundary, subject to the homogeneous Dirichlet boundary condition, is exactly the interval  $(\lambda_1(2), \infty)$ , where  $\lambda_1(2)$  stands for the principal frequency of the Laplace operator in  $\Omega$ .

**Acknowledgements:** This research has been partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2016-2233.

**Two parameter singular perturbation problems for abstract second order differential equations with lipschitzian nonlinearities**

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**Abstract.** Let  $H$  be a real Hilbert space endowed with the scalar product  $(\cdot, \cdot)$  and the norm  $|\cdot|$ , and  $V$  be a real Hilbert space endowed with the norm  $\|\cdot\|$ . Let  $A : V \subset H \rightarrow H$ , be a linear self-adjoint operator and  $B$  is nonlinear  $A^{1/2}$  lipschitzian operator. Consider the following Cauchy problem:

$$\begin{cases} \varepsilon u''_{\varepsilon\delta}(t) + \delta u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) + B(u_{\varepsilon\delta}(t)) = f(t), & t \in (0, T), \\ u_{\varepsilon\delta}(0) = u_0 & u'_{\varepsilon\delta}(0) = u_1, \end{cases} \quad (2)$$

where  $u_0, u_1, f : [0; T] \rightarrow H$  and  $\varepsilon, \delta$  are two small parameters.

Using theorems of existence and uniqueness of solutions to the problems (2), (3); a priori estimates of these solutions and a relationship, obtained in [1], between solutions to the problem for the abstract linear second order differential equation and the corresponding solution to the problem for the first order equation, we establish convergence estimates for the difference of solutions  $u_{\varepsilon\delta}$  to the problem (2) and the solution to the problem:

$$\begin{cases} \delta l'_\delta(t) + Al_\delta(t) + B(l_\delta(t)) = f(t), & t \in (0, T), \\ l_\delta(0) = u_0; \end{cases} \quad (3)$$

in the case when  $\varepsilon \rightarrow 0$  and  $\delta \geq \delta_0 > 0$  or

$$Av(t) + B(v(t)) = f(t), \quad t \in (0, T), \quad (4)$$

in the case when  $\varepsilon \rightarrow 0$  and  $\delta \rightarrow 0$ .

#### Reference:

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#### On a family of torsional creep problems in Finsler metrics

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**Abstract.** The asymptotic behavior of solutions to a family of Dirichlet boundary value problems involving differential operators in divergence form on a domain equipped with a Finsler metric is investigated. Solutions are shown to converge uniformly to the distance function to the boundary of the domain which takes into account the Finsler norm involved in the equation. This implies that a well-known result in the analysis of problems modelling torsional creep continues to hold in this more general setting. This is based on a joint work with M. Făcășanu and M. Mihăilescu. This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P1-1.1-TE-2019-0456.

#### Controllability properties of the heat equation with a memory term

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**Abstract.** This article studies the controllability properties of the heat equation having a memory term with higher order derivatives. It is known that in the usual setting the diffusion equation with a memory term has poor controllability properties. We show that these difficulties can be overcome if a moving control is considered.

#### On the positive solutions of a fractional thermostat model

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**Abstract.** We study the existence of positive solutions of a nonlinear fractional heat equation with nonlocal boundary conditions depending on a positive parameter. Our results extend the second-order thermostat model to the non-integer-order case. We base our analysis on the known Guo-Krasnosel'skii fixed point theorem on cones. At the end, one example is also presented to support our result.

**Keywords:** Thermostat model, fixed point, Guo-Krasnosel'skii fixed point theorem.

**Subject Classification:** 30D05, 39B52, 47H10.

**Asynchronous Combinational Systems****Șerban VLAD**

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**Abstract.** The Boolean asynchronous systems are the dynamical systems which are generated by the functions  $\Phi : \{0, 1\}^n \times \{0, 1\}^m \rightarrow \{0, 1\}^n$  whose coordinates  $\Phi_i, i \in \{0, \dots, n\}$  are computed independently on each other. In the special case when these systems have no feedback loops, they are called combinational (we refer also to combinational circuits). Our purpose is to define the asynchronous combinational systems, and then to address some properties, such as: stability, hazards, isomorphisms, invariance and basins of attraction.

**Stability for systems of 1-D coupled nonlinear oscillators****Cristian VLADIMIRESCU**

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**Abstract.** The stability of the null solution of different systems of differential equations describing the motion of 1-D coupled nonlinear oscillators is discussed. Under certain assumptions we derive some stability results, which are in agreement with physical reality.

This is joint work with Professor Gheorghe Moroșanu (Babeș-Bolyai University, Romania)

**2010 Mathematics Subject Classification:** 34D20.**Key words and phrases:** coupled oscillators, uniform stability, asymptotic stability.**Haar Wavelet Collocation Method for Linear First Order Stiff Differential Equations****Mehmet Tarik ATAY, Onur Metin MERTASLAN, Musa Kasim AGCA, Abdulkadir YILMAZ, and Batuhan TOKER**

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**Abstract.** In general, there are countless types of problems encountered from different disciplines that can be represented by differential equations. These problems can be solved analytically in simpler cases; however, computational procedures are required for more complicated cases. Right at this point, the wavelet-based methods have been using to compute these kinds of equations in a more effective way. The Haar Wavelet is one of the appropriate methods that belongs to the wavelet family using to solve stiff ordinary differential equations (ODEs). In this study, The Haar Wavelet method is applied to stiff differential problems in order to demonstrate the accuracy and efficacy of this method by comparing the exact solutions. In comparison, similar to the exact solutions, the Haar wavelet method gives adequate results to stiff differential problems.

**Kirchhoff problems with lack of compactness****Youpei ZHANG**

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**Abstract.** We are concerned with the existence of infinitely many solutions for a class of nonlocal Kirchhoff problems with lack of compactness. Our arguments combine variational, topological and analytical methods. This talk is based on our recent work

Y. Zhang, X. Tang, D. Qin, Infinitely many solutions for Kirchhoff problems with lack of compactness. *Nonlinear Anal.* 197 (2020), 111856, 31 pp.